4E1213

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B. Tech. IV - Sem. (Main) Exam., May - 2019

BSC Computer Sc. & Engg.

4CS2 – 01 Discrete Mathematics Structure

CS, IT

Time: 3 Hours **Maximum Marks: 120**

Instructions to Candidates:

Attempt all ten questions from Part A, five questions out of seven questions from Part B and four questions out of five from Part C.

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A

(Answer should be given up to 25 words only) **[10×2=20]**

All questions are compulsory

Q.1 Prove that for any two sets A and B: $A - (A \cap B) = A - B$

Q.2 Give an example of a partially ordered set which is not a lattice.

Q.3 Show that the multiplicative group $G = \{1, -1, i, -i\}$ is cyclic. Find its generators.

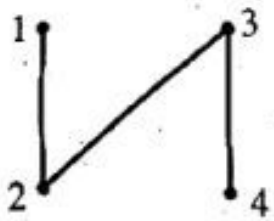
Q.4 Define finite state Machines.

Q.5 Find the minimum number of students in a school to be sure that 5 of them are born in the same month.

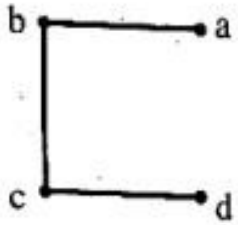
Q.6 Prove that α^2 is an even integer, then α is an even integer.

Q.7 Find the generating function for the sequence $\{1, 1, 0, 0, 1, 1, 1, \ldots\ldots \infty\}$

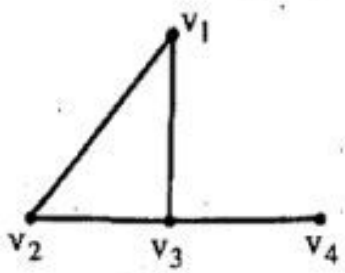
Q.8 Prove that these graphs G_1, G_3 and G_2, G_3 are non – isomorphic.



(G_1)



(G_2)



(G_3)

Q.9 Find the domain of the following function:

$$f(x) = \sqrt{\log\left(\frac{5x-x^2}{4}\right)}$$

Q.10 In how many ways can a team of 11 cricketers be chosen for 6 bowlers, 4 wicket keepers and 11 batsman to give majority of batsman so that at least 4 bowlers are there and 1 wicketkeeper?

PART – B

(Analytical/Problem solving questions) **[5×8=40]**

Attempt any five questions

Q.1 (a) Write the scope and objective of DMS in Computer Science? [4]

(b) In a test 70% of the candidates passed in Science, 65% in Mathematics, 27% failed in both Science and Mathematics and 124 passed in both the subjects. Find the total number of candidates for the test. [4]

Q.2 Show that in the power set P(A) of all subsets of a set A = {a, b, c}, 'Set inclusion, $\subseteq$' is a partial order relation. Also draw the Hasse diagram for the POSET. [6+2=8]

Q.3 (a) Solve the recurrence relations – [4]

$a_n - 5a_{n-1} + 6a_{n-2} = 3n^2 - 2n + 1$

(b) Prove by induction that sum of the cubes of three consecutive integers is divisible by 9. [4]

Q.4 (a) Let $f : R \to R$ and $g: R \to R$ where R is the set of real numbers. Find gof and fog where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective or bijective. [4]

(b) Draw the transition diagram of the finite state machine M (I, S, O, s_0, f, g), where $I = \{a, b\}$, $S = \{S_0, S_1\}$, $O = \{0, 1\}$ and the transition table is as follows – [4]

	f		g	
S \ I	a	b	a	b
s_0	s_1	s_0	0	1
s_1	s_0	s_1	1	0

Also, find the output string for the input b b a a.

Q.5 Define and explain the following by suitable examples – [4×2=8]

(i) Cyclic group (ii) Order of an element in a group

(iii) Field (iv) Zero divisor of a ring

Q.6 (a) *Show that* $\sim (pv (\sim p \wedge q)) \equiv (\sim p) \wedge (\sim q)$ (without truth table) [4]

(b) Write contrapositive converse and inverse of the statement "The home team wins whenever it is raining". Also construct the truth table for each statement. [4]

Q.7 Write short notes on the following – http://www.rtuonline.com [4×2=8]

(a) Planar graphs

(b) Isomorphism of graphs

(c) Cut sets

(d) Vertex connectivity

PART – C

(Descriptive/Analytical/Problem Solving/Design Questions) **[4×15=60]**

Attempt any four questions

Q.1 Let $R = \{(1, 2), (2, 3), (3, 1)\}$ and $A = \{1, 2, 3\}$. Find reflexive, symmetric and transitive closure of R using – [5×3=15]

(a) Composition of relation R

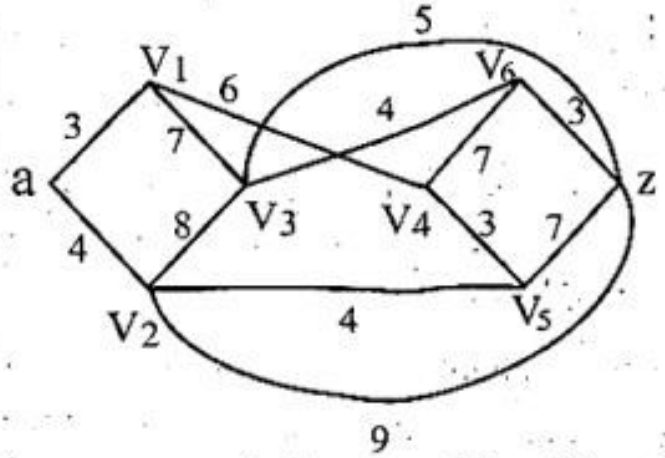
(b) Composition of matrix relation R

(c) Graphical representation of R

Q.2 (a) Define Bounded lattices, complement of an element of a lattices and distributive lattices. [6]

(b) Let $(L, \leq)$ be a bounded distributive Lattice, if an element $a \in L$, has a complement then it is unique. [9]

Q.3 (a) Find the shortest path from a to z in the following graph – [5]



(b) Suppose that a connected planar graph has 30 vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph. [5]

(c) Let G be the set of all non – zero real numbers and Let $a * b = \frac{ab}{2}$, then show that $(G, *)$ is an abelian group. [5]

Q.4 (a) Obtain the Principal disjunctive normal forms of $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$. [5]

(b) Let $\Delta(G)$ be the maximum of the degrees of the vertices of a graph G then $K(G) \leq 1 + \Delta(G)$ where K(G) is the chromatic number of graph. [5]

(c) In a complete graph with n – vertices there are $\frac{(n-1)}{2}$ edge disjoint Hamiltonian circuits, if n is an odd number ≥ 3. [5]

Q.5. (a) Define tautology and prove the following – [4]

$\{(p \rightarrow q) \wedge p\} \rightarrow q$ is tautology

(b) Define fallacy and prove the following – [4]

$(p \wedge q) \vee \sim(p \wedge q)$ is $\wedge$ fallacy

(c) Let (m, *) be a semi group and $a \in m$ such that the equations $a * u = x$ and $v * a = x$ have solutions in M for all $x \in M$. Show that (M, *) is a monoid. [7]