3E1616

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B. Tech. (Sem. III) (Main/Back) Examination, December 2017

Applied Elect. & Inst. Engg.

3AI1 Mathematics III (EC, EIC, BM, AI, CR, PE, PC)

Time : 3 Hours **Maximum Marks : 80**

Min. Passing Marks : 26

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used / calculated must be stated clearly.*

Use of following supporting materials is permitted during examination. (Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) Find Laplace transform of the function $\sin\sqrt{x}$ and hence or otherwise obtain Laplace transform of $\frac{\cos\sqrt{x}}{\sqrt{x}}$.

8

(b) Find the inverse Laplace transform of $\log\sqrt{1+\frac{9}{s^2}}$.

8

OR

1 (a) Use Laplace transform theory to solve the following equation :

$\left(D^2+1\right)y = x\cos x$, where $y=0$, $\frac{dy}{dx}=0$ at $x=0$.

8

(b) State and prove convolution theorem for Laplace transform.

8

UNIT - II

2 (a) Obtain the Fourier series of the function $f(x) = x - x^2$, $-\pi < x \leq \pi$ and deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots.$$

8

(b) In the Z transform, show that $Z\left\{n^p\right\} = -z\frac{d}{dz}Z\left\{n^{p-1}\right\}$, and hence evaluate $Z\left\{n^3\right\}$.

8

OR

2 (a) If $f(x) = \begin{cases} x & 0 \leq x \leq \pi/2 \\ \pi - x, & \pi/2 < x \leq \pi \end{cases}$, then find half range cosine series of $f(x)$.

8

(b) Using convolution theorem, find the inverse Z transform of $\frac{z^2}{\left(z^2 - 4z + 3\right)}$.

8

UNIT - III

3 (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate the integral $\int_0^\infty \frac{x\cos x - \sin x}{x^3}\cos\frac{x}{2}\,dx$.

8

(b) Using Fourier transform, find the solution of the initial boundary value problem.

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \; -\infty < x < \infty, \; t > 0$$

$u(x, 0) = f(x)$, $u(x, t)$ is finite as $x \to \pm\infty$.

8

OR

3 (a) Find the Fourier sine and cosine transform of the function x^{m-1}.

8

(b) Find $f(x)$, if its Fourier sine transform is, $\frac{p}{(1+p)^2}$.

8

UNIT - IV

4 (a) If $f(z) = u + iv$ is an analytic function of $z = x + i\,y$ and

$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$ then, find u and v and the corresponding analytic function $f(z)$.

6

(b) Find the bilinear transformation which maps the points $z = \infty$, i, 0 into the points $w = 0$, i, ∞.

6

(b) Prove that $\int_C \frac{dz}{(z-a)} = 2\pi i$, where C is given by the equation $|z - a| = R$.

4

OR

4 (a) Show that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^2 y^5 (x + iy)}{x^4 + y^{10}}, \; z \neq 0, \; f(0) = 0$$

is not analytic at the origin although Cauchy Riemann equations are satisfied at the origin.

6

(b) Evaluate the integral $\int_C \frac{e^{2z}dz}{(z+a)^4}$, where C is the circle $|z|=3$. 6

(c) Show that function $u = \cos x \cosh y$ is harmonic and find its harmonic conjugate. 4

UNIT - V

5 (a) Expand $f(z)=\frac{1}{(z-1)(z-3)}$ in the power of z which are valid for regions:

(i) $|z|<1$

(ii) $1<|z|<3$

(iii) $|z|>3$ 8

(b) Use method of contour integration to prove that

$$\int_0^{2\pi} \frac{d\theta}{1+a^2-2a\cos\theta} = \frac{2\pi}{1-a^2};\ 0<a<1..$$ 8

OR

5 (a) Find the residues of $\frac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1,2,3$ and ∞, and show that their sum is zero. 8

(b) Use method of contour integration to evaluate $\int_0^{\infty} \frac{\cos mx}{a^2+x^2}dx$. 8