

**Instructions to Candidates:**

Attempt all ten questions from Part A, five questions out of seven from Part B and four questions out of five from Part C. ✓

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly. ✓

Use of following supporting material is permitted during examination. (Mentioned in form No. 205) 205

1. NIL

2. NIL

**PART - A**

(Answer should be given up to 25 words only)

[10×2=20] ✓

All questions are compulsory

Q.1 Construct the forward difference table for the function  $f(x) = \tan x$  for  $0.10 \leq x \leq 0.30$

by taking  $h = 0.5$ .

Q.2 Prove that  $E = e^{hD}$ , where symbols have their usual meanings.

Q.3 Write Gauss forward and Gauss backward interpolation formula.

Q.4 What is numerical integration formula in Simpson's 3/8 rule?

Q.5 Using Runge-Kutta second order method, the approximate solution of the differential

equation  $\frac{dy}{dx} = f(x, y)$ ;  $y(x_0) = y_0$  is given by

$y_{n+1} = y_n + 2\alpha(k_1 + k_2)$ , where  $k_1 = hf(x_n, y_n)$  and  $k_2 = hf(x_n + \beta h, y_n + \gamma k_1)$ . Then

what are the values  $\alpha, \beta, \gamma$ ?

Q.6 State existence condition of Laplace Transform.

Q.7 Find Laplace transform of  $b^x f(ax)$ .

Q.8 State convolution theorem for Fourier transforms.

Q.9 Write damping rule for z-transform.

Q.10 Write a function whose z-transform is equal to 1.

## PART - B

(Analytical/Problem solving questions)

[5x8=40]

Attempt any five questions

Q.1 Prove that

$$u_1 x + u_2 x^2 + u_3 x^3 + \dots = \frac{x}{1-x} u_1 + \left(\frac{x}{1-x}\right)^2 \Delta u_1 + \left(\frac{x}{1-x}\right)^3 \Delta^2 u_1 + \dots$$

Q.2 Using Lagrange's interpolation formula, find the value of  $\log_{10} 301$  for the following data -

	$x_0$	$x_1$	$x_2$	$x_3$
$x$	300	304	305	307
$\log_{10} x = f(x)$	2.477	2.482	2.484	2.4871

Q.3 Evaluate  $\sqrt{28}$  to 4 decimal places by Newton-Raphson method.

Q.4 Using Runge - Kutta method, obtain a solution of the equation

$$\frac{dy}{dx} = xy; y(1) = 2$$

for  $x = 1.4$ , using  $h = 0.2$ .

Q.5 Define Dirac Delta Function and find its Laplace and Fourier transforms.

Q.6 Find the Fourier transform of  $f(x)$  defined by -

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

And hence evaluate  $\int_{-\infty}^{\infty} \frac{\sin sa \cos x}{s} ds$ .

Q.7 Using convolution theorem, evaluate

$$Z^{-1} \left\{ \frac{z^2}{z^2 - 1} \right\}$$

### PART - C

(Descriptive/Analytical/Problem Solving/Design Questions) [4×15=60]

Attempt any four questions

Q.1 (a) Find inverse Laplace transform of  $s \log \left( \frac{s-1}{s+1} \right) + 2$ .

(2) (b) Using Newton - Gregory forward formula, find interpolation polynomial, which passes through the points (1, -1), (2, -1), (3, 1) and (4, 5).

Q.2 (a) Given that  $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$  and  $y(0) = 1$ ,  $y(0.1) = 1.06$ ,  $y(0.2) = 1.12$ ,  $y(0.3) = 1.21$ . Evaluate  $y(0.4)$  by Milne's predictor method.

(b) Find the value of  $\log_e 2$  from  $\int_0^1 \frac{x^2}{1+x^3} dx$ . using Simpson's  $\frac{1}{3}$  rule by dividing the range into five ordinates.

Q.3 (a) Use Laplace transform theory to solve the initial value problem

$$\frac{dy}{dt} + y = f(t), y(0) = 2, \text{ where } f(t) = \begin{cases} 0, & 0 \leq t < \pi/2 \\ \cos t, & t \geq \pi/2 \end{cases}$$

(b) Use Stirling formula to find  $y_{23}$  given:

$$y_{20} = 49225, y_{25} = 48316, y_{30} = 47236, y_{35} = 45926, y_{40} = 44306.$$

Q.4 (a) Find the complex Fourier transforms of  $e^{-bt}$ .

(b) Using Regula Falsi method find real root of equation  $x^2 + 4 \sin x = 0$ .

Q.5 (a) Find  $f(x)$  if its Fourier cosine transform is  $\frac{1}{1+s^2}$ . ✓

(b) Using Z - transform solve the difference equation  $6u_{n+2} - u_{n+1} - u_n = 0$ , given that  $u(0) = 0, u(1) = 1$ .