

4E4131

Roll No. _____

Total No. of Pages : 7

4E4131

B. Tech. IV-Sem. (Main) Exam; April-May 2017
Electronics & Communication Engg.
4EC2A Random Variables & Stochastic Processes

Time : 3 Hours

Maximum Marks : 80

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Min. Passing Marks : 24

Instructions to Candidates :-

Attempt any five questions, selecting one question from each unit. All Questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.

Units of quantities used calculated must be stated clearly.

Use of following supporting material is permitted during examination.

(Mentioned in form No. 205)

1. NIL2. NIL**UNIT - I**

1 (a) State the theorem of total probability and Baye's theorem on inverse probability.

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(b) For a certain binary communication channel, the probability that a transmitted '0' is received as '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (i) a '1' is received (ii) a '1' was transmitted given that '1' was received.

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OR

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1

[P.T.O.

- 1 (a) A fair dice is rolled 5 times. Find the probability that 1 shows twice, 3 shows twice and 6 shows once.

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- (b) State Bernoulli's theorem on independent trials.

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- (c) Each of two persons A and B tosses 3 fair coins. What is the probability that they obtain the same number of heads ?

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UNIT - II

- 2 (a) A random variable (continuous) 'x' has a pdf $f(x) = k x^2 e^{-x}; x \geq 0$. Find k, mean and variance.

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- (b) If a continuous random variable 'x' has Rayleigh density

$$f(x) = \frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}} \text{Xv}(x), \text{ find } E(X^n) \text{ and deduce the values of}$$

$$E(X) \text{ and } \text{var}(X).$$

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OR

- 2 (a) If 'x' is $N(\mu; \sigma^2)$ then show that $z = \frac{(x-\mu)}{\sigma}$ is a standard normal random variable; that is $N(0, 1)$.

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- (b) A noisy transmission channel has a per digit error probability $P_e=0.01$.
(i) Calculate the probability of more than one error in 10 received digits.
(ii) Repeat using Poisson approximation.

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- (c) The pdf of a random variable x is given by

$$f_x(x) = \begin{cases} k & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

Calculate $P((x) \leq c)$ for $c = \frac{1}{2}$ if $a = -11, b = 2$.

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UNIT - III

- 3 (a) The joint pdf of (x, y) is given by $f(x, y) = 24xy; x > 0, y > 0, x + y \leq 1$ and $f(x, y) = 0$, elsewhere, find the conditional mean and variance of y given x.

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- (b) State central limit theorem and verify it for random variables (independent)

$$x_k, \text{ where for each } k, P(x_k = \pm 1) = \frac{1}{2}.$$

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OR

- 3 (a) Calculate the correlation coefficients for the following ages of husband (x) and wives (y).

x:	23	27	28	28	29	30	31	33	35	36
y:	18	20	22	27	21	29	27	29	28	29

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- (b) Let x and y be defined by

$$x = \cos \Theta \text{ and } y = \sin \Theta$$

where Θ is a random variable uniformly distributed over $[0, 2\pi]$.

- (i) Show x and y are uncorrelated
(ii) Show that x and y are not independent.

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UNIT - IV

- 4 (a) Consider a random process $x(t)$ is defined by

$$x(t) = u \cos t + v \sin t \quad -\infty < t < \infty$$

where 'u' and 'v' are independent random variables, each of which assumes the values -2 and 1 with the probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Show that $x(t)$ is wss but not strict sense stationary.

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- (b) Verify the equations :

(i) $R_{xy}(-\tau) = R_{yx}(\tau)$

(ii) $|R_{xy}(\tau)| \leq \sqrt{R_x(0)R_y(0)}$

(iii) $|R_{xy}(\tau)| \leq \frac{1}{2} [R_x(0) + R_y(0)]$.

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OR

- 4 (a) If the input to a continuous time linear system is a random process $\{X(t), t \in T_x\}$ and output of the linear system is $\{Y(t), t \in T_y\}$.

Find the autocorrelation function of $Y(t)$.

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(b) If $X(t)$ is a wss random process and has a m.s. derivative $X'(t)$ then show that

$$(i) R_{XX'}(\tau) = \frac{d}{d\tau} R_x(\tau)$$

$$(ii) R_{X'}(\tau) = -\frac{d^2}{d\tau^2} R_x(\tau).$$

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UNIT - V

5 (a) A wss random process $X(t)$ with autocorrelation function

$R_x(\tau) = e^{-a|\tau|}$ where 'a' is a real positive constant is applied to the input of an LTI system with impulse response $h(t) = e^{-bt}u(t)$ find the autocorrelation function of the output $Y(t)$.

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(b) A zero mean wss random process is called band limited white noise if its spectral density is given by

$$S_X(\omega) = \begin{cases} N_0/2 & |\omega| \leq \omega B \\ 0 & |\omega| > \omega B \end{cases}$$

Find the autocorrelation function of $X(t)$.

8

OR

- 5 (a) Let $Y(t)$ be the output of an LTI system with impulse response $n(t)$ when a wss random process $X(t)$ is applied as input. Show that

(i) $S_{xy}(w) = n(w)S_x(w)$

(ii) $S_y(w) = n^*(w)S_{xy}(w)$

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- (b) Consider a wss process $X(t)$ with autocorrelation function $R_x(z)$ and power spectral density $S_x(w)$ let $X'(t) = dx(t)/dt$ show that $S_{x'}(w) = w^2 S_x(w)$.

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